CSCE 235 **Exam 1** **Sep 29, 2000**
Name : **SS#**

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Instructions: Do all of the following questions. Show your work and explain your answers.

Question 1: (20 points) Let $A = \{2\}$, $B = \{2, 3\}$, and $X = \{1, 2, 3, 4\}$. Find

(a) $A \setminus B$.

(b) The power set of $A \cup B$.

(c) A^c.

(d) $A \times B$.

Question 2: (10 points) Use truth tables to prove that:

$$(a \implies b) \text{ is equivalent to } (\sim a \vee b). \text{ (i.e. } (a \implies b) \equiv (\sim a \vee b).)$$

Question 3: (15 points) Let $A = \{1, 2, 3\}$ and define the following equivalence relation, R, on A:

$$R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}.$$

(i) Is R antisymmetric? Explain.

(ii) Find the equivalence class of 3.

Question 4: (15 points) Decide whether the following statements are true or false (i.e. prove or disprove). If they are false, then give a counter example.

(a) If n is an **even** integer, then **3n-5** is an even integer.

(b) For any sets A and B, if $A \subseteq B$, then $A^c \subseteq B^c$.

Question 5: (10 points) Use mathematical induction to prove the following:

$$1 + 2 + 2^2 + 2^3 + ... + 2^n = 2^{n+1} - 1, \text{ for } n \geq 1.$$

Question 6: (20 points)

(a) Give an example of a bijective function from the set of even natural numbers $2\mathbb{N}$ to the set of odd natural numbers $2\mathbb{N} - 1$.

(b) Prove that the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = e^x + 1$ is bijective (i.e one-to-one and onto.)

Question 7: (10 points) Draw the Hasse diagram of the poset $(A, \subseteq)$, where

$$A = \{\{1\}, \{1, 2\}, \{1, 2, 3\}, \{1, 2, 4\}\}.$$

Extra Credit:

(1) Let $A = \{1, 2, 3\}$.

(a) (7 points) Give an example of a binary relation on A which is both symmetric and antisymmetric.

(b) (7 points) Give an example of a binary relation on A which is not symmetric and not antisymmetric.

(2) (7 points) Give an example of a bijective function from $(\mathbb{N} \setminus \{1, 2\})$ to $\mathbb{N}\setminus]\{1\}$.