CSCE 235 **Sep 29, 00**

Exam #1

Name: SS#

Do **all** of the following questions and show your work. Explain each step. **Do NOT use calculators.**

Question 1:

(a) (20 points) Give a bijective function between the set $(\mathbb{N} \cup \{0\}) \times \{0, 1\})$ and the set $\mathbb{N} \cup \{0\}$.

(b) (10 points) Let the function f from $(\mathbb{N} \cup \{0\}) \times (\mathbb{N} \cup \{0\})$ to $2(\mathbb{N} \cup \{0\})$ be defined by:

$$f(n, k) = 2^k(2n + 1) - 1.$$

Show that f is onto.

Question 2:

(a) (14 points) Define the following binary relation on $\mathbb{N} \times \mathbb{N}$:

$$(a, b) \sim (c, d) \text{ if and only if } a + d = b + c.$$

Show that $\sim$ is transitive.

(b) (14 points) Let $A = \{\{1\}, \{\sqrt{2}\}, \{3\}\}$. Give an example of a binary relation on A which is not symmetric and not antisymmetric.

Question 3:

(a) (14 points) Let $p \in \mathbb{N}$, $p \geq 624$, and let $w \in \mathbb{R}$, $w \neq -1$. Use mathematical induction to prove that:

$$w^p + w^{p+1} + ... + w^{p+n-1} + 2 = 2 + \tfrac{w^p - w^{p+n}}{1-w}, \ \forall n \in \mathbb{N}..$$

(b) (14 points) Find the following sum. Do not use calculators.

$$2^{5553} + 8 + 16 + 32 + 64 + 128 + ... + 2^{5552}.$$

(c) (14 points) Solve the following recurrence relation:

$$9a_n = 6a_{n-1} - a_{n-2},\ n \geq 2, \text{ given } a_0 = 3,\ a_1 = -1.$$