Generalized centro-invertible matrices

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Abstract

Centrosymmetric matrices are those complex matrices $A \in \mathbb{C}^{n\times n}$ such that $AJ_n = J_n A$ where $J_n \in \mathbb{C}^{n\times n}$ is the matrix with 1's on the secondary diagonal and 0's otherwise. An important property of centrosymmetric matrices says that if A is a centrosymmetric matrix with γ linearly independent eigenvectors, then γ linearly independent eigenvectors of A can be chosen to be symmetric or skew-centrosymmetric. The particular case of matrices that commute with a permutation matrix was studied in [11] by Stuart and Weaver. This class of matrices has been widely studied considering their applications [3, 12, 13]. In [1], Abu-Jeib has used them to analyze some spectral properties of regular magic squares. After applying the Sinc collocation method to Sturm-Liouville Problems, the resulting matrices are centrosymmetric. Eigenspectrum properties of symmetric centrosymmetric matrices presented in [5] are applied for solving a generalized eigensystem of smaller dimensions than the original ones. Moreover, algorithms for solving centrosymmetric linear systems of equations are presented in [4]. Not only direct problems have been solved by using centrosymmetric matrices, but also the inverse eigenproblem and its approximation have been considered by Bai in [2]. By generalizing centrosymmetric matrices, centrohermitian matrices are defined as those matrices $A \in \mathbb{C}^{m\times n}$ satisfying $J_m A J_n = \overline{A}$, where $\overline{A}$ means the conjugate of the corresponding entries of A. Each square centrohermitian matrix is similar to a matrix with real entries and full information about the spectral properties of square centrohermitian matrices is given for instance in [10].

On the other hand, a more general situation where the equation $KA = A^{s+1}K$ is studied for involutory matrices K has been analyzed in [7, 8] for nonnegative integer values of s. They are called $\{K, s+1\}$-potent matrices and when $s = 0$ it is called a $\{K\}$-centrosymmetric matrix. Complementing this study, the case $s = -2$, that corresponds to generalized centro-invertible matrices, has been considered in [9]. In addition, some applications for image blurring/deblurring were developed. Clearly, $\{K, s+1\}$-potent matrices generalize centrosymmetric ones to a wider class of matrices.

In addition, the sign function is studied in [6]. It is observed that in general, sign(A) is a nonprimary square root of the identity. However, the square root can be obtained from the sign function. It is also shown that the sign function has applications in several areas. For example, it plays an important role in

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iterative methods for matrix roots and the polar decomposition. Moreover, it is useful in applications to Riccati equations, the eigenvalue problem, and lattice quantum chromodynamics. Moreover, it is well-known that every square matrix A is $\{sign(A)\}$-centrosymmetric.

This paper presents a deeper study of generalized centro-invertible matrices. First of all, we state the coordinability between the classes of involutory matrices, generalized centro-invertible matrices, and $\{K\}$-centrosymmetric matrices. Then, some characterizations of generalized centro-invertible matrices are obtained. A spectral study of generalized centro-invertible matrices is given. In particular, we prove that the sign of a generalized centro-invertible matrix is $\{K\}$-centrosymmetric. Finally, some algorithms have been developed for the construction of generalized centro-invertible matrices.

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